Name:

Spring 2016 Math 245 Final Exam

Please read the following directions:
Please write legibly, with plenty of white space. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. You may use a single page of notes, but no calculators or other aids. This exam will last 120 minutes; pace yourself accordingly. Please try to keep a quiet test environment for everyone. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
11.	5		10
12.	5		10
13.	5		10
14.	5		10
15.	5		10
16.	5		10
17.	5		10
18.	5		10
19.	5		10
20.	5		10
Total:	100		200

Problem 1. Carefully define the following five terms:

a. proposition

b. tautology

c. contrapositive

d. conjunctive addition

e. conjunctive simplification

Problem 2. Carefully define the following four terms:

a. direct proof

b. vacuous proof

c. proof by contradiction

d. proof by induction

Problem 3. Carefully define the following terms:

a. reflexive

b. symmetric

c. transitive

d. total order

e. well order

Problem 4. Carefully define the following terms:

a. graph

b. loop

c. simple graph

d. bipartite graph

e. forest

Problem 5. Compute the number of edges in $K_{m,n}$.

Problem 6. How many 4 digit numbers have no two consecutive repeated digits?

Problem 7. Find a simple strongly connected digraph on 4 vertices with 5 edges.

Problem 8. A graph has an 8×8 adjacency matrix A. Suppose A has the special property that each row sum is 5. Name two things you can conclude from this special property.

Problem 9. Prove or disprove: $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, \exists z \in \mathbb{R}, (x-z)^2 < (x-y)^2$.

Problem 10. Let $a, b, c \in \mathbb{Z}$. Prove that if $a|b$ and $a|c$, then $a|(b+3c)$.

Problem 11. Suppose that $A \subseteq B \subseteq C$. Prove that $(A \times B) \subseteq (B \times C)$.

Problem 12. Solve the recurrence given by $a_0 = 1, a_1 = 4, a_n = 2a_{n-1} - a_{n-2}$ $(n \geq 2)$.

Problem 13. Set $A = \{1, 2\}$, $B = \{3, 4, 5\}$. Compute the number of relations from A to B.

Problem 14. My sock drawer contains 10 black socks, 12 grey socks, and 2 hot pink socks. All 24 socks are identical except for color. I reach in without looking and pull out a handful of socks. What is the smallest number I need in my handful to ensure at least one matching pair of socks? Be sure to prove your answer.

Problem 15. Draw an unbalanced complete binary tree with 7 vertices.

Problem 16. Find all spanning trees in K_4, and compute how many there are.

Problem 17. Compute the number of Hamiltonian circuits in $K_{3,3}$.

Problem 18. Use the binomial theorem to find the coefficient of x^{16} in $(x^2 - 2)^{10}$.

Problem 19. Use the binomial theorem to calculate and simplify the first three terms of the series for $\sqrt[3]{x+1}$.

Problem 20. Let $f : X \to Y$ and $g : Y \to Z$ be functions. Suppose $g \circ f$ is injective. Prove that f must be injective.